1993:

1) $f(x) = x^3 - 5x^2 + 3x + k$

a) $f'(x) = 3x^2 - 10x + 3$

$f'(x) = 0$

$3x^2 - 10x + 3 = 0$

$x = \frac{1}{3}, 3$

$f'(x)$: $+$ | $1/3$ | $-$ | 3 | $+$

Increasing on $(-\infty, 1/3)$ and $(3, \infty)$

b) $f''(x) = 6x - 10$

$f''(x) = 0$

$6x - 10 = 0$

$6x = 10$

$x = \frac{5}{3}$

$f''(x)$: $-$ | $5/3$ | $+$

concave downward on $(-\infty, 5/3)$

c) Rel. min when $x = 3$ since $f'(x)$ changes sign from $-$ve to +

$f(3) = 3^3 - 5(3^2) + 3(3) + k = 11$

$27 - 5(9) + 9 + k = 11$

$k = 11 + 9$

$k = 20$

2) $x(t) = 2te^{-t}, \ t \geq 0$

a) $V(t) = 2e^{-t} + 2t(-1)e^{-t}$

$= 2e^{-t} - 2te^{-t}$

$a(t) = -2e^{-t} - (2e^{-t} + 2t(-1)e^{-t})$

$= -2e^{-t} - 2e^{-t} + 2te^{-t}$

$= -4e^{-t} + 2te^{-t}$

$a(0) = -4e^0 + 0 = -4$

b) When $a(t) = 0$

$-4e^{-t} + 2te^{-t} = 0$

$2e^{-t}(-2 + t) = 0$

$e^{-t} \neq 0, \quad -2 + t = 0$

$t = 2$

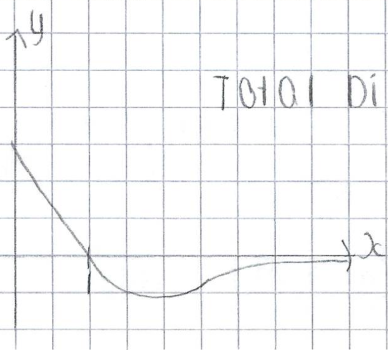
$V(2) = 2e^{-2} - 2(2)e^{-2}$

$= 2e^{-2} - 4e^{-2}$

$= -2e^{-2}$

$= -\frac{2}{e^2}$

~~b)~~ **c)** $V(t) = 2e^{-t}(1 - t)$



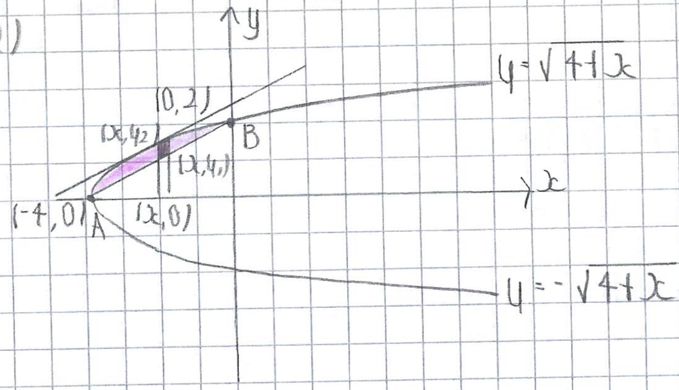
Total Distance $= \int_0^1 V(t)\,dt - \int_1^5 V(t$

$= \left[2te^{-t}\right]_0^1 - \left[2te^{-t}\right]_1^5$

$= 2e^{-1} - 0 - (10e^{-5} -$

$= \frac{2}{e} - \frac{10}{e^5} + \frac{2}{e} = \frac{4}{e} - \frac{10}{e^5}$

3) $y^2 = 4 + x$, $A(-4,0)$, $B(0,2)$
$y = \pm\sqrt{4+x}$



a) Mean value theorem:

$$f'(c) = \frac{f(0) - f(-4)}{0-(-4)}$$
$$\frac{1}{2}(4+x)^{-1/2} = \frac{2-0}{4}$$
$$\frac{1}{2}(4+x)^{-1/2} = \frac{1}{2}$$
$$(4+x)^{-1/2} = 1$$
$$1 = \sqrt{4+x}$$
$$4+x = 1$$
$$x = -3 \rightarrow y = +\sqrt{1}$$

$(-3, 1)$

b) $$\text{Area } R = \int_{-4}^{0}\left(\sqrt{4+x} - \frac{x+4}{2}\right)dx$$
$$= \left[\frac{2}{3}(4+x)^{3/2} - \frac{x^2}{4} - 2x\right]_{-4}^{0}$$
$$= \frac{2}{3}(4)^{3/2} - 0 - 0 - (0 - 4 + 8)$$
$$= \frac{16}{3} - 4$$
$$= \frac{4}{3}$$

Equation AB: $y - 0 = \frac{0-2}{-4-0}(x+4)$
$y - 0 = \frac{1}{2}(x+4)$
$2y = x+4$
$y = \frac{x+4}{2}$

c) $R(x) = y_2 - 0 = \sqrt{4+x}$
$r(x) = y_1 - 0 = \frac{x+4}{2}$

$$\text{Volume} = \pi\int_{-4}^{0}\left((4+x) - \left(\frac{x+4}{2}\right)^2\right)dx$$
$$= \pi\left[4x + \frac{x^2}{2} - \frac{(x+4)^3}{12}\right]_{-4}^{0}$$
$$= \pi\left[0 + 0 - \frac{64}{12} - (-16 + 8 - 0)\right]$$
$$= \pi\left(8 - \frac{64}{12}\right)$$
$$= \frac{32}{12}\pi$$
$$= \frac{8}{3}\pi$$

4) $f(x) = \ln(2 + \sin x)$, $\pi \le x \le 2\pi$

a) $f'(x) = \frac{\cos x}{2 + \sin x}$
$f'(x) = 0$
$\frac{\cos x}{2+\sin x} = 0$
$\cos x = 0$ $\quad 2 + \sin x = 0$
$x = \frac{3\pi}{2}$ $\quad \sin x \neq -2$

$f'(x)$: π $-$ $\frac{3\pi}{2}$ $+$ 2π

$f\left(\frac{3\pi}{2}\right) = \ln(2-1) = \ln 1 = 0$

Rel. min at $x = \frac{3\pi}{2}$ since $f'(x)$ changes sign from -ve to +ve point extrema:

$f(\pi) = \ln(2+0) = \ln 2$

$f(2\pi) = \ln(2+0) = \ln 2$

Absolute min $= 0$

Absolute max $= \ln 2$

b) $f''(x) = \dfrac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2}$

$= \dfrac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$

$= \dfrac{-2\sin x - 1}{(2+\sin x)^2}$

$f''(x) = 0$

$\dfrac{-2\sin x - 1}{(2+\sin x)^2} = 0$

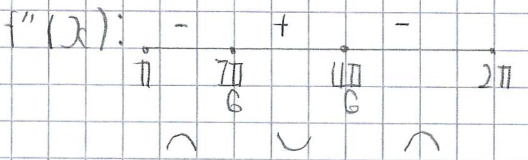
$-2\sin x - 1 = 0$ $\qquad$ $2+\sin x = 0$

$2\sin x = -1$ $\qquad$ $\sin x \neq -2$

$\sin x = -\dfrac{1}{2}$

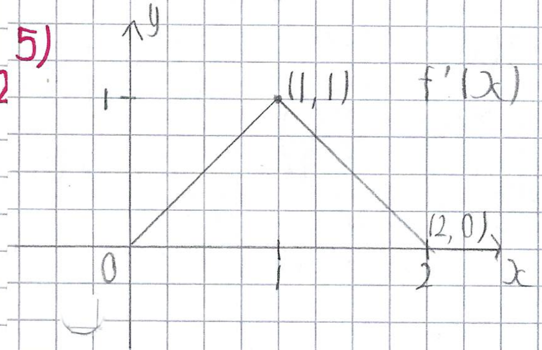
$x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$f''(x)$:



sign: points of inflexion occur at $x = \dfrac{7\pi}{6}, x = \dfrac{11\pi}{6}$

5) 2



a) For $0 < x < 1$: $y - 0 = 1(x-0)$

$y = x$

For $1 \leq x < 2$: $y - 0 = \dfrac{0-1}{2-1}(x-2)$

$y - 0 = -1(x-2)$

$y = -x + 2$

$y = 2 - x$

$f'(x) = \begin{cases} x & , 0 < x < 1 \\ 2 - x & , 1 \leq x < 2 \end{cases}$

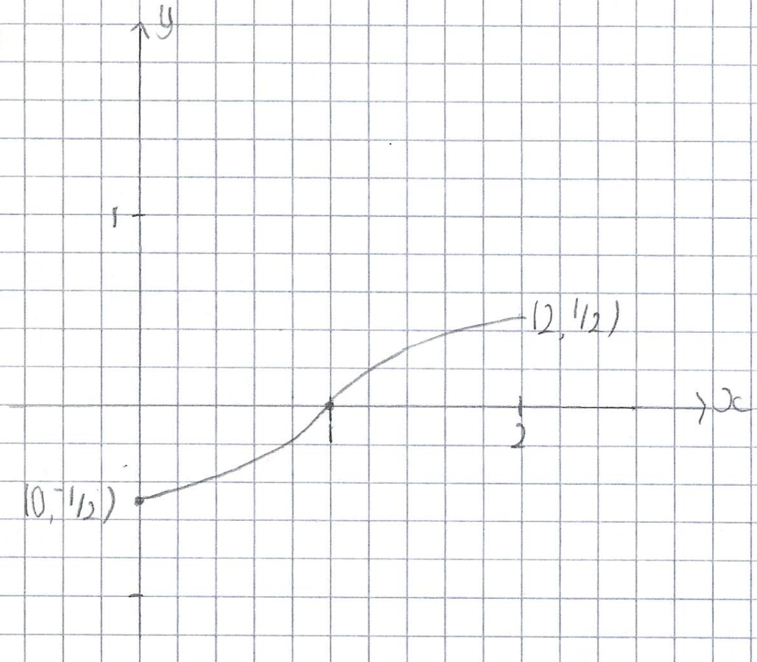
b) $f(x) = \begin{cases} x^2/2 + c_1 & , 0 < x < 1 \\ 2x - x^2/2 + c_1 & , 1 \leq x < 2 \end{cases}$

since $f(1) = 0$

$\dfrac{1}{2} + c_1 = 0 \Rightarrow 1 + 2c_1 = 0 \Rightarrow c_1 = -\dfrac{1}{2}$ and $2 - \dfrac{1}{2} + c_1 = 0 \Rightarrow c_1 = -\dfrac{3}{2}$

$$f(x) = \begin{cases} \frac{x^2}{2} - \frac{1}{2} & , 0 < x < 1 \\ -\frac{x^2}{2} + 2x - \frac{3}{2} & , 1 \le x < 2 \end{cases}$$

c)



6) $\frac{dP(t)}{dt} \propto 800 - P(t)$

$\frac{dP(t)}{dt} = k(800 - P(t))$

a) $P(0) = 500$

$\int \frac{dP(t)}{800 - P(t)} = \int k\,dt$

$-\ln(800 - P(t)) = kt + c$

$(800 - P(t))^{-1} = e^{kt+c}$

$\frac{1}{800 - P(t)} = Ce^{kt}$

$\frac{1}{800 - 500} = Ce^0$

$C = \frac{1}{300}$

$\frac{1}{800 - P(t)} = \frac{1}{300} e^{kt}$

$e^{kt} = \frac{300}{800 - P(t)}$

$(800 - P(t))e^{kt} = 300$

$800 - P(t) = 300e^{-kt}$

$P(t) = 800 - 300e^{-kt}$

b) $P(2) = 700$

$700 = 800 - 300e^{-2k}$

$\frac{300}{e^{2k}} = 100$

$300 = 100e^{2k}$

$e^{2k} = 3$

$2k = \ln 3$

$k = \frac{\ln 3}{2}$

c) $P(t) = 800 - 300e^{(-\ln 3/2)t}$

$\lim_{t \to \infty} P(t) = \lim_{t \to \infty} \left(800 - \frac{300}{e^{\infty}}\right)$

$= 800$